Compaction-Induced Deformation on Thin Layer of Flexible Substrate for Medical Devices

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1. Introduction

The design of medical devices is a complex task and it involves the resolution of conflicts and compromise among the desired feature. One of a more inclusive feature is any substance or combination of substances synthetic or natural in origin, which can be used for any period of time, as a whole or part of a system which treats, augments or replace tissue, organ or function of the body. This definition must be extended because biomaterials are currently being utilized as scaffolds for tissue-engineered devices (hybrids of synthetic or biologic scaffolds and living cells and tissue for vessels, heart valves, and myocardium).

For example, the cardiovascular biomaterials may contact blood (both arterial and venous), vascular endothelial cells, fibroblasts, as well as a number of other cells and acellular matrix material that make up all biological tissue. At the same time, orthopedic biomaterials are complex and important role of the cellular component of bone. Then the general properties of bone, ligament, tendon, and cartilage have increased substrate stiffness under the compaction indenter and it initiates stress concentration. Therefore, it can be concluded that it is essential to achieve effective compaction on thin layer of flexible substrate.

Keywords : Deformation, Flexible substrate, Biomaterials

Abstract

This paper describes the methodology for evaluation of compaction-induced stresses and deformation on thin layer of flexible substrate by using finite element analysis. The incremental placement and compaction of thin layer of flexible substrate are based on a hysteretic model for residual stresses induced by multiple cycles of loading and unloading. The results showed that the large compaction load can be applied to thin layer of flexible substrate and it achieves higher density effectively. The reinforcement of layer also increases compaction efficiency, because it reduces the ratio between shear and vertical forces during compaction process. The maximum vertical stress on the base of specimen usually decreased with higher compaction thickness. The reinforcement will bw...
behavior of large grain ensembles. This approach has proven to be successful in understanding the bulk granular material response induced by various impact conditions [3].

The present study investigates the effectiveness of a composite reinforcement on thin layer of flexible substrate during compaction. Other simulation concepts like the finite element method mostly are used to investigate the phenomena of substrate layer interaction under quasi-static condition like constant loading. So, it is necessary to take into account the important influence of the layer on the dynamic behavior of substrate. Furthermore, the dynamical layer-load-changes enlarge, loading to an increased deformation of the surface. The following axle of the substrate will be excited by the back unevenness of the first axle, which again induces the excitation of the substrate.

2. Materials and Methods

A finite element model have been developed for analysis of stresses and deformations. The resulting from placement and compaction of layers of fill simulates the actual sequence of field operations in a number of sequential steps. These analytical procedures permit modeling of multiple cycles of compaction loading at any given fill stage with a single solution increment. In these analyses, compaction loading is realistically considered as a transient moving concentrated surficial load which may pass more times over some specified portion of the surface. In Figure 1 shows a spatially periodic loading (of period L) is applied to the surface of a horizontally layered profile of an elastic layer.

Therefore, a strip loading as a spatially periodic loading is applied to the surface of a horizontally layered profile of a flexible substrate. It is well known for such a periodic loading (or loading function). A Fourier series representation can be used Nemat-Nasser [4]. For instance, if the Cartesian coordinate in the horizontal direction is $x$ and the loading function
The loading has been represented by the sum of periodic functions (in this case cosine functions because the loading function \(p(x)\) was chosen to be an even function of \(x\)). It may be observed that, for such a spatially periodic loading, the displacements in the substrate below are also periodic. It is therefore possible to write the displacements as a series of periodic functions [5].

\[
\begin{align*}
    u_x(x, z) &= \sum_{n=0}^{\infty} U^{(n)}(z) \sin \alpha_n x \\
    u_z(x, z) &= \sum_{n=0}^{\infty} W^{(n)}(z) \cos \alpha_n x
\end{align*}
\]  

(3.1) 

(3.2)

However, the equations (2.1 and 2.2) will be used in situations in which no confusion arises. Suppose that we now take one term of the cosine series representing the loading function \(p(x)\), and obtain the solution to the problem associated with this single sinusoidal load applied to the surface of the substrate layer.

\[
p(x) = P_n \cos \alpha_n x
\]

(4.1) 

Then

\[
\begin{align*}
    u_x &= U_n(z) \sin \alpha_n x \\
    u_z &= W_n(z) \cos \alpha_n x
\end{align*}
\]  

(4.2)

At the same time, we will be assumed that conditions of plane strain prevail so that there is no displacement in the \(y\) direction and no variation of the field quantities with \(y\). Then it is only necessary to consider stress and displacement fields having the form

\[
\begin{align*}
    u_x &= U(\alpha, z) \sin \alpha x \\
    u_z &= W(\alpha, z) \cos \alpha x \\
    \sigma_{xz} &= T(\alpha, z) \sin \alpha x \\
    \sigma_{zz} &= N(\alpha, z) \cos \alpha x \\
    \sigma_{xy} &= H(\alpha, z) \cos \alpha x \\
    \sigma_{yy} &= M(\alpha, z) \cos \alpha x
\end{align*}
\]  

(5.1) 

(5.2) 

(5.3) 

(5.4) 

(5.5) 

(5.6)

Since the solution for more complex load cases can be obtained by superposition of components given by equations (5) and where it has been assumed that \(\alpha\) stands for any particular value of \(\alpha_n = 2n\pi / L\) and \(U^{(n)} = U(\alpha_n, z)\), etc.
Let us now consider the determination of the field quantities in a particular layer, \( l \) of the material \([6]\). If equations (5) are substituted into the equilibrium equations it is found that

\[
-\alpha H + \frac{\partial T}{\partial z} = 0 \quad (6.1)
\]

\[
-\alpha T + \frac{\partial N}{\partial z} = 0 \quad (6.2)
\]

Similarly if equations (5.1-5.6) are substituted into Hooke’s Law it is found that

\[
\alpha U = \frac{1}{E_i} \left[ H - \nu_i (M + N) \right] \quad (7.1)
\]

\[
\frac{\partial W}{\partial z} = \frac{1}{E_i} \left[ N - \nu_i (H + M) \right] \quad (7.2)
\]

\[
\frac{\partial U}{\partial z} - \alpha W = \frac{2(1+\nu_i)T}{E_i} \quad (7.3)
\]

\[M = \nu_i (H + N) \quad (7.4)\]

where \( E_i, \nu_i \) denote the values of Young’s Modulus and Poisson’s ratio for layer, \( l \).

Equations (6), (7) allow the non-zero stress and displacement to be expressed in terms of \( N \) and thus

\[
H = -\frac{\partial^2 N}{\partial Z^2} \quad \text{and} \quad T = -\frac{\partial N}{\partial Z}
\]

\[M = \nu_i \left( N - \frac{\partial^2 N}{\partial Z^2} \right) \quad (8)\]

\[
\alpha E^* U = -\frac{\partial^3 N}{\partial Z^3} - \nu^* N
\]

\[
\alpha E^* W = -\frac{\partial^3 N}{\partial Z^3} + (2 + \nu^*)N
\]

where \( E^* = \frac{E_i}{1-\nu_i^2}, \nu^* = \frac{\nu_i}{1-\nu_i} \)

and \( Z = \alpha z \)

and \( \frac{\partial^4 N}{\partial Z^4} - 2 \frac{\partial^2 N}{\partial Z^2} + N = 0 \quad (9.1) \)

If equation (9.1) is now solved, we obtain

\[N = X_1 C + X_2 Z + X_3 S + X_4 Z C \quad (9.2)\]

The fundamental step in the finite layer technique is to determine the four constants \( X_1, \ldots, X_4 \) in terms of boundary quantities. To be more precise, suppose that layer \( l \) is bounded by the node planes \((z = z_l)\) and \((z = z_m)\) where \( m = l + 1 \) and that the subscripts \( l, m \) indicate the value of a particular quantity on the indicated node plane. Then the solution can be used to determine \( X_1, \ldots, X_4 \) in terms of \( U_l, W_l, U_m, W_m \). Once \( X_1, \ldots, X_4 \) are known they can be used to evaluate \( T_l, N_l, T_m, N_m \) and so establish a relationship of the form

\[
\begin{bmatrix}
-T_l \\
-N_l \\
+T_m \\
+N_m
\end{bmatrix}
= \begin{bmatrix}
k_{11} & k_{12} & k_{13} & k_{14} \\
k_{21} & k_{22} & k_{23} & k_{24} \\
k_{31} & k_{32} & k_{33} & k_{34} \\
k_{41} & k_{42} & k_{43} & k_{44}
\end{bmatrix}
\begin{bmatrix}
U_l \\
W_l \\
U_m \\
W_m
\end{bmatrix}
\]

\[
(10)
\]

\[
\text{Figure 2 Single cell layer within the surface of flexible substrate.}
\]

\[
\text{Figure 3 Non-homogeneous profile tested with exponential approximation (adapted from [6]).}
\]
The matrix occurring in equation (10) is called the layer stiffness matrix of the particular layer (layer, \( l \) in this case). Layer stiffness matrices can be used to construct solutions for layered deposits in exactly the same way as element stiffness matrices are used in conventional applications of the finite element method.

Consequently, we present a novel way to study the behavior of layer of flexible material when it received forces by compression. Thus FEM technique is necessary to use a dynamic solving algorithm where not only the displacements of the nodes, but as coming from the Newton’s dynamic equilibrium the accelerations \( \ddot{u} \) for each node in the model are considered as Hiroma et al.\[7\]

\[
\ddot{u}(i) = M^{-1}(P(i) - I(i)) 
\]

where \( M \) is the mass matrix, \( P \) is the vector of the external forces (i.e. contact forces or nodal forces of neighboring elements) and \( I \) is the vector of internal forces on the basis of the stress inside the elements.

The mass matrix can be obtained by distributing the mass of each element on to its nodes. The created point mass belonging to a node is the sum of all partial masses of all the elements defined at this node. Using the explicit solving algorithm, available with ANSYS, the velocities \( \dot{u} \) and the displacements \( u \) for all degrees of freedom can be calculated directly from the information at the beginning of the time increment. The velocities and the displacements were integrated over the time according the following formulas.

\[
\dot{u}(i+\frac{1}{2}) = \frac{\Delta t}{2}(\dot{u}(i)+\dot{u}(i+1)) 
\]

\[
u(i+1) = u(i)+\Delta t \left( \dot{u}(i+\frac{1}{2}) \right)
\]

The material property of the flexible substrate in this simulation corresponds to granule with the following parameters: cohesion, \( c = 0.029 \text{ N/mm}^2 \); angle of internal friction, \( \phi = 22.5^\circ \); density, \( \rho = 2.06 \text{ t/mm}^3 \); elasticity, \( E = 20 \text{ N/mm} \). The dynamic behavior of the flexible substrate is condensed on the damping by the energy loss of the plastic deformation and on the effect of mass-inertia of substrate elements in this simulation.

3. Results and Discussion

Results from such an analysis are shown in Figure 4 and 5. A circular and a rectangular loading have been considered. Each loading is applied to the surface of a substrate layer which is made up of two sublayers; sublayer (A) is of depth \( H_A \) while the lower layer (B) is of depth \( H_B \). The loadings were chosen so that they had the same minimum dimension, i.e. the loading, \( q \) was applied over the region \( |x| < a \) (strip), \( 0 < r \leq a \) (circle), \( |x| < a, |x| < b \) (rectangular). The material was assumed to be anisotropic. For layer A, \( E_h / E_v = 1.5, G / E_v = 0.45, \nu_h = 0.25, \nu_{hv} = 0.2 \) and for layer B, \( E_h / E_v = 33, G / E_v = 0.5, \nu_h = 0.1, \nu_{hv} = 0.9, \nu_{vh} = 0.3 \) (The subscripts \( h, v \) indicate horizontal and vertical directions, respectively.).

![Figure 4: Surface pressure mobilization of single cell on the surface of flexible substrate.](image)

The plots shown in Figure 5 is for the vertical \( \sigma_{zz} \) stresses along the axis of the loading (\( x = y = r = 0 \)). There is a large difference in the vertical stress computed for each of the loading types. However, there is less difference in the horizontal stresses for this particular substrate profile.

![Figure 5: Stresses in a substrate consisting of two anisotropic layers.](image)
The plots shown in Figure 5 is for the vertical $zz$ stresses along the axis of the loading ($x = y = r = 0$). There is a large difference in the vertical stress computed for each of the loading types. However, there is less difference in the horizontal stresses for this particular substrate profile.

All of the problems discussed thus far have been concerned with vertical loading, however there are many engineering problems which involve lateral or horizontal loadings applied to foundations. An example of study is shown the result in Figure 6, where a uniform shear, it is shown applied to the surface of a thin layered flexible substrate over a circular region of radius, $a$. The variation of vertical, $u_z$ and horizontal $u_x$ displacements with depth is calculated for the three layered system shown in Figure 6 ($x/a = 0.5, y/a = 0$). For this experiment the material properties of each of the sublayers A, B, C are described below:

Layer A: $E_h/E_v = 2, G_v/E_v = 0.4, h_A = 0.3, h_B = 0.2$
Layer B: $E_h/E_v = 2, G_v/E_v = 0.4, h_A = 0.3, h_B = 0.2$
Layer C: $E_h/E_v = 1, G_v/E_v = 1/3, h_A = 0.5, h_B = 0.5$

and the ratio of Young’s modulus in the layers is assumed to be

$$(E_v)_A : (E_v)_B : (E_v)_C = 25 : 5 : 1$$

Figure 7 shows the time-settlement behavior of strip loading on a substrate layer with a shear modulus which increases with depth (non-dimensional time, $\theta$ where $\theta = c_0 t / \ell$).

The coefficient of consideration, $c_0$ is defined on the figure in terms of the Poisson’s ratio of the substrate, and its permeability per unit weight of water $k / w$. It may be observed from the time settlement plots that as the rate of increase of shear modulus with depth $\ell_m / \ell$ becomes larger the final deflections become smaller and occur at smaller time, $\theta$.

Problems involving the time-dependent deformation of materials under a constant applied loading may often be treated by assuming that the materials display viscoelastic behavior, i.e. that the material properties are themselves dependent on time. In this study, we used the assumption that the behavior of the material may be separated into a deviatoric and a volumetric behavior. That is to say there will be a different time-dependent response to a mean stress increase, then to a deviatoric or shear stress increased.

Figure 8 shows an example of the relation of deflection vs time for central point of a strip or circular loading resting on a layered material. It involves loading applied to a flexible substrate which consists of an upper viscoelastic layer and a lower elastic layer. Both materials were assumed to have a constant Poisson’s ration, $\nu$ that of the upper layer being $\nu = 0.5$ (i.e. incompressible) and that of the lower layer $\nu = 0.3$. It
was assumed that the deformation of the material was due to a time-dependent shear modulus.

4. Conclusions

In the paper a two dimensional FEM of the compaction-induced deformation on thin layer of flexible substrate was presented. With this extension to the simulation concept, the new concept based on the dynamical FE-simulation was developed, that includes the possibility to investigate the dynamical load changes, caused by vertical oscillation of indenter and the according substrate deformation. It was pointed out, that the indenter oscillation takes influence to the mobility of the substrate. Due to these oscillations, the dynamical compaction on thin layer of substrate changes enlarge, leading to an increased deformation of the surface, which again induces the excitation of the substrate. Therefore, dynamic loads and substrate deformation influence mutually.

References